Basic *Mathematica* Commands

Defining and Evaluating Functions

To define the function $f(x) = x^2 + 2$, type and evaluate the following cell.

```
f[x_] := x^2 + 2
```

Note that no output is generated, and that the colon and underscore are necessary for function definition **but not for function evaluation**. You could also use a template to enter the exponent. Templates are also useful for quotients and exponential functions.

$$g[x\_] := E^{3x} + \frac{Sin[x]}{Cos[2x] + 4}$$

To evaluate a function at a given value, type the function's name and its argument in square brackets.

```
f[2.7]
```

```
9.29
```

```
g[-5]
```

$$\frac{1}{e^{15}} - \frac{Sin[5]}{4 + Cos[10]}$$

```
g[-5.]
```

```
0.303368
```

Notice that if you include a decimal in your input, you receive decimal output. If not, you receive exact output.

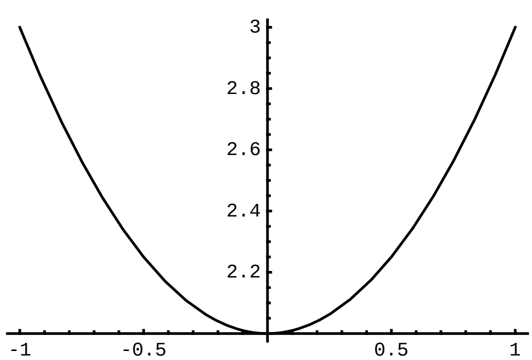
Plotting Graphs

To plot the graph of $f(x)$ over the interval $a \le x \le b$, we use the following command.

```
Plot[f[x], {x, a, b}]
```

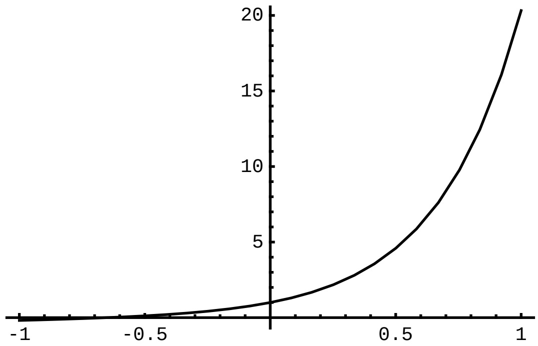
To plot f and g as defined above over the interval $-1 \le x \le 1$, we type and evaluate the following cell.

```
Plot[f[x], {x, -1, 1}]
```




```
- Graphics -
```

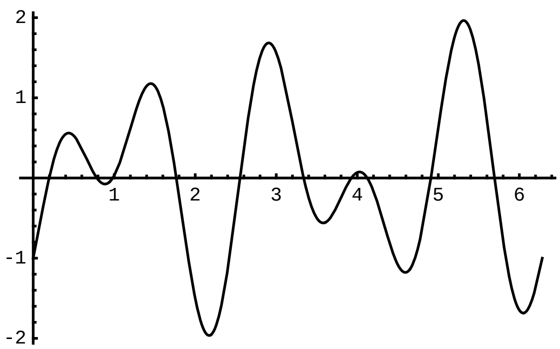
```
Plot[g[x], {x, -1, 1}]
```



- Graphics -

To plot a function you haven't already defined, just type the function definition into the command. The semi-colon at the end of the command suppresses the word "Graphics".

```
Plot[Sin[5 t] - Cos[3 t], {t, 0, 2 π}]
```



Clearing Definitions

To remove all previous definitions of a variable *x* or a function *f*, type the following.

```
Clear[x]
```

```
Clear[f]
```

Solving Equations

To solve an equation for a variable *x*, we use the following command. **The two equal signs are very important!**

```
Solve[f[x] == g[x], x]
```

Here are a few examples.

```
Solve[x^2 - 4 x + 5 == 0, x]
```

{{x → 2 - ⅈ}, {x → 2 + ⅈ}}

```
Solve[3 t - 1 == 2 t + 3, t]
```

{{t → 4}}

```
Solve[a x - b t == c x^2 - t, x]
```

$$\left\{\left\{x\to\frac{a-\sqrt{a^2+4\,c\,t-4\,b\,c\,t}}{2\,c}\right\},\left\{x\to\frac{a+\sqrt{a^2+4\,c\,t-4\,b\,c\,t}}{2\,c}\right\}\right\}$$

We can also solve two equations simultaneously.

```
Solve[{y == 3 x - 2, 3 y == 4 x - 4}, {x, y}]
```

$$\left\{\left\{x\to\frac{2}{5},\ y\to-\frac{4}{5}\right\}\right\}$$

The Solve command is useful for finding precise solutions to equations. However, this often this is not desirable or even possible. Consider the following equation.

```
Solve[x^3 + 3 x^2 - 4 x - 1 == 0, x]
```

$$\left\{\left\{x\to-1+\frac{\left(\frac{1}{2}\left(-45+i\sqrt{2091}\right)\right)^{1/3}}{3^{2/3}}+\frac{7}{\left(\frac{3}{2}\left(-45+i\sqrt{2091}\right)\right)^{1/3}}\right\},\right.$$

$$\left\{x\to-1-\frac{\left(1+i\sqrt{3}\right)\left(\frac{1}{2}\left(-45+i\sqrt{2091}\right)\right)^{1/3}}{2\times3^{2/3}}-\frac{7\left(1-i\sqrt{3}\right)}{2^{2/3}\left(3\left(-45+i\sqrt{2091}\right)\right)^{1/3}}\right\},$$

$$\left.\left\{x\to-1-\frac{\left(1-i\sqrt{3}\right)\left(\frac{1}{2}\left(-45+i\sqrt{2091}\right)\right)^{1/3}}{2\times3^{2/3}}-\frac{7\left(1+i\sqrt{3}\right)}{2^{2/3}\left(3\left(-45+i\sqrt{2091}\right)\right)^{1/3}}\right\}\right\}$$

Compare this solution to the following. Note that the only difference is the decimal point on one coefficient.

```
Solve[x^3 + 3. x^2 - 4 x - 1 == 0, x]
```

$\{\{x\to-3.94883\},\ \{x\to-0.217184\},\ \{x\to1.16601\}\}$

What about this one?

```
Solve[Cos[x] == x, x]
```

```
Solve::tdep : The equations appear to involve
   the variables to be solved for in an essentially non-algebraic way.
```

```
Solve[Cos[x] == x, x]
```

The Solve command doesn't work. However, we can use FindRoot to approximate a solution. The number 1 is an initial guess that we usually obtain from a graph.

```
FindRoot[Cos[x] == x, {x, 1}]
```

$\{x\to0.739085\}$

Built-in Functions and Constants

Mathematica has many built-in functions. All of them require square brackets around the argument. Here are a few examples.

```
Sin[x]  Cos[x]  Exp[x]  ArcTan[x]  Sqrt[x]
```

$e^x\sqrt{x}$ ArcTan[x] Cos[x] Sin[x]

The function Exp[x] is another way of writing E^x. *Mathematica* also has every mathematical constant you can imagine, all beginning with a capital letter.

```
Pi   E  GoldenRatio
```

e GoldenRatio π

Calculus

To find a derivative, use the D command or put a prime after the function's name.

```
D[x^3 - 4, x]
```

$3 x^2$

```
g'[x]
```

$$3 e^{3 x} + \frac{\text{Cos}[x]}{4 + \text{Cos}[2 x]} + \frac{2 \text{Sin}[x] \text{Sin}[2 x]}{(4 + \text{Cos}[2 x])^2}$$

To find a second derivative, use the D command as shown or put two primes after the function's name.

```
D[x^3 - 4, {x, 2}]
```

$6 x$

```
g''[x]
```

$$9 e^{3 x} - \frac{\text{Sin}[x]}{4 + \text{Cos}[2 x]} + \frac{4 \text{Cos}[x] \text{Sin}[2 x]}{(4 + \text{Cos}[2 x])^2} + \text{Sin}[x] \left(\frac{4 \text{Cos}[2 x]}{(4 + \text{Cos}[2 x])^2} + \frac{8 \text{Sin}[2 x]^2}{(4 + \text{Cos}[2 x])^3}\right)$$

Definite and indefinite integrals are evaluated as shown.

```
Integrate[Sin[3 x], x]
```

$$-\frac{1}{3} \text{Cos}[3 x]$$

```
Integrate[Sin[3 x], {x, -1, 2}]
```

$$\frac{\text{Cos}[3]}{3} - \frac{\text{Cos}[6]}{3}$$

Numerical integration is sometimes needed as well.

```
Integrate[E^(x^2), {x, 0, 2}]
```

$$\frac{1}{2} \sqrt{\pi} \text{ Erfi}[2]$$

```
NIntegrate[E^(x^2), {x, 0, 2}]
```

16.4526

Creating Lists of Values

To create a list of numbers or function outputs, use the command below.

```
Table[h[i], {i, 1, 5}]
```

{h[1], h[2], h[3], h[4], h[5]}

```
Table[h[i], {i, -1, 3, .5}]
```

```
{h[-1], h[-0.5], h[0.], h[0.5], h[1.], h[1.5], h[2.], h[2.5], h[3.]}
```

```
Table[n^2 - 1, {n, 1, 10}]
```

```
{0, 3, 8, 15, 24, 35, 48, 63, 80, 99}
```

Evaluating Sums

To evaluate sums, you need to specify a function and a variable with a range and a step size. If no step size is indicated, *Mathematica* assumes a step size of 1.

```
Sum[i^2, {i, 1, 25}]
```

```
5525
```

```
Sum[i^2, {i, 0, 25, 1 / 2}]
```

$$\frac{42925}{4}$$

```
Sum[i^2, {i, 0, 25, .5}]
```

```
10731.3
```

Lists

To enter a list of points, use curly brackets around each point and around the entire list.

```
examplelist = {{1, 2}, {4, 2}, {5, 3}, {7, 1}}
```

```
{{1, 2}, {4, 2}, {5, 3}, {7, 1}}
```

You can also use the Table command.

```
newlist = Table[{i, i^2}, {i, 0, 3, .5}]
```

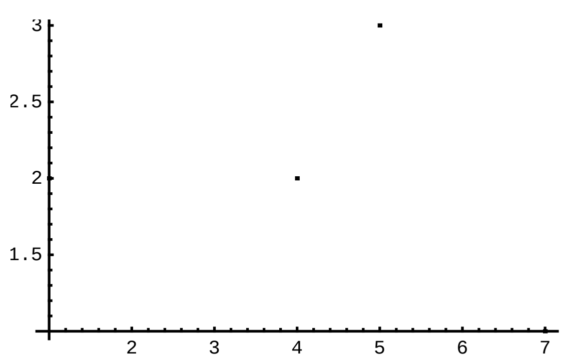
```
{{0, 0}, {0.5, 0.25}, {1., 1.}, {1.5, 2.25}, {2., 4.}, {2.5, 6.25}, {3., 9.}}
```

A semicolon will suppress the output.

```
examplelist = {{1, 2}, {4, 2}, {5, 3}, {7, 1}};
```

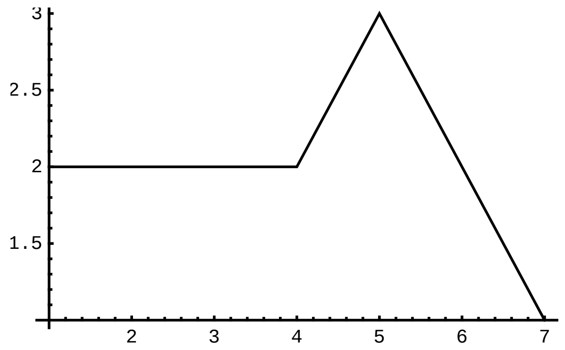
To plot a list, use the command below.

```
ListPlot[examplelist]
```



There are a number of options you can specify.

```
ListPlot[examplelist, Joined → True]
```



List Manipulations

Multiplying two lists is done one coordinate at a time. Notice that this is NOT matrix multiplication, nor is it the dot product from vector calculus!

```
{1, 2, 3, 4} {a, b, c, d}
```

{a, 2 b, 3 c, 4 d}

```
2 {a, b, c, d}
```

{2 a, 2 b, 2 c, 2 d}

```
2 + {a, b, c, d}
```

{2 + a, 2 + b, 2 + c, 2 + d}

```
f[x_] := x^2
```

```
f[{a, b, c, d}]
```

$\{a^2, b^2, c^2, d^2\}$